Chiral vacuum alignment in dense QCD

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Abstract

We discuss an interesting possibility of nontrivial, quark-mass induced chiral vacuum alignment in color-flavor locking phase of cold, dense QCD. With the simplifying assumption that the gaps for quarks are identical to those of antiquarks and the light quark masses are given by $m_u = m_d$ and $m_s/m_d = 15$, we find the true chiral vacuum can align only to one of the discrete number of directions in the continuum of chiral vacua. The alignment depends on the size of the diquark condensates, and the vacuum transitions between the discrete vacua caused by the evolution of the diquark condensates can be first order phase transition with vanishing or nonvanishing latent heat, depending on the vacua involved. It is also shown that as $\mu \to \infty$, where μ is the baryon chemical potential, parity is spontaneously broken through the vacuum alignment.

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The chiral symmetry $SU_L(3) \times SU_R(3)$ of quantum chromodynamics (QCD) with three massless quark flavors is spontaneously broken to $SU_{L+R}(3)$ at low energies. Any point in the continuum of the chiral vacua, the coset space $[SU_L(3) \times SU_R(3)]/SU_{L+R}(3)$, can be chosen as the vacuum, since any two vacua are equivalent as far as physics is concerned. However when quarks receive small (current) masses the chiral symmetry is then explicitly broken, and the degeneracy of the vacua is lifted. The quark mass term then picks up the lowest energy state in the continuum of the vacua as the true chiral vacuum. This is called Dashen's chiral vacuum alignment [1]. To determine the true vacuum one has to find the quark-mass induced potential that lifts the degeneracy of the vacua. Since the Nambu-Goldstone bosons of the chiral symmetry breaking are small fluctuations about a point in the continuum of the vacua, the potential is nothing but the meson mass term in the chiral Lagrangian of the nonlinearly realized Nambu-Goldstone bosons. Therefore, the true vacuum is the one that minimizes the potential

$$V(\Sigma) = -\mathcal{L}_m(\Sigma) \propto -\text{Re}[\text{Tr}(m\Sigma)], \tag{1}$$

where $\Sigma \in SU(3)$ denotes the chiral fields for the Nambu-Goldstone bosons, while $\mathcal{L}_m(\Sigma)$ is the quark-mass induced meson mass term in the chiral Lagrangian.

When the true vacuum is nontrivial, that is, $\Sigma_0 \neq I$, where Σ_0 is the vacuum that minimizes the potential, some interesting phenomena could arise. For instance, if the quark mass matrix were given as $m \propto \text{Diag}(-1, -1, -\delta)$ with $\delta > 1/2$, then the vacuum Σ_0 would have an imaginary part in its matrix elements [1]. Then writing $\Sigma = \Sigma_0 \exp(i\Pi^A T^A)$, where Π^A and T^A denote the octet pseudo scalar mesons and the SU(3) generators, respectively, it can be easily seen that the mass term (1) could give rise to a spontaneous CP violation, which enables, for example, η decay into two pions. Of course, with the quark mass matrix realized in nature, the true vacuum is at $\Sigma_0 = I$, and there is no such CP violation.

Recently, cold, dense quark system with large baryon chemical potential μ has received strong interest [2,3]. As well known, the Fermi surface of such a dense system is unstable against Cooper pairing when an attractive force is present. At large μ a dressed gluon

exchange (in hard dense loop approximation) provides such a force, and causes the system to be in color-flavor locking phase in which quarks condense in a pattern [2]

$$\langle \chi_i^a \chi_j^b \rangle \propto k_1 \, (U_0)^a{}_i (U_0)^b{}_j + k_2 \, (U_0)^a{}_j (U_0)^b{}_i$$

$$\langle \bar{\varphi}_i^a \bar{\varphi}_j^b \rangle \propto -[k_1 \, (V_0)^a{}_i (V_0)^b{}_j + k_2 \, (V_0)^a{}_j (V_0)^b{}_i], \tag{2}$$

where χ_i^a, φ_i^a, with a the color index, $i = 1, \cdots, 3$, the flavor index, denote the two-component Weyl fermions for the left-handed quarks and the complex conjugate of the right-handed quarks, respectively. The unitary matrices $U_0, V_0 \in \mathrm{U}(3)$ can be arbitrary in the absence of axial U(1) anomaly, but the anomaly effect, even though small because of the suppression of instanton effects at high density [4,5], chooses a parity even vacuum [5,6] in which

$$U_0 = V_0. \tag{3}$$

Upon the condensation of the quarks the symmetry of dense, massless QCD, $\mathrm{SU}_L(3) \times \mathrm{SU}_R(3) \times \mathrm{U}_A(1) \times \mathrm{U}_B(1)$, where $\mathrm{U}_A(1)$ and $\mathrm{U}_B(1)$ denote the axial and the baryon number symmetry, respectively, is spontaneously broken to $\mathrm{SU}_{L+R}(3)$, generating 10 Nambu-Goldstone bosons (mesons). The $\mathrm{U}_A(1)$ is not an exact symmetry, but at large μ, due to the suppression of anomaly, can be regarded a good *approximate* symmetry. As in vacuum QCD a continuum of chiral vacua $[\mathrm{SU}_L(3) \times \mathrm{SU}_R(3) \times \mathrm{U}_A(1) \times \mathrm{U}_B(1)]/\mathrm{SU}_{L+R}(3)$ arises upon the spontaneous symmetry breaking. Note, however, that the vacua connected by the $\mathrm{U}_A(1)$ rotation are only approximately degenerate, because the $\mathrm{U}_A(1)$ is not an exact symmetry. It turns out that the vacuum of lowest energy is parity even, hence the relation (3).

Now, when the current quark masses are turned on, the degeneracy of the chiral vacua is lifted. The true vacuum can be picked up, as in vacuum QCD, by minimizing the quark mass induced potential $V(\Sigma) = -\mathcal{L}_m(\Sigma)$ of dense QCD.

The meson mass term $\mathcal{L}_m(\Sigma)$ in color-flavor locking phase has a different form than in vacuum QCD because of the absence of left-right quark condensates. For small quark masses, $\mathcal{L}_m(\Sigma)$ can be expanded in powers of the quark mass matrix. The absence of a left-right quark condensate due to the suppression of instanton effects renders the leading term

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to be quadratic in quark mass [2]. The most general form for the leading $\mathcal{L}_m(\Sigma)$, consistent with the chiral symmetry and the condensates (2), is given by [7–11]

$$\mathcal{L}_m(\Sigma) = A \ [\text{Tr}(m^t \Sigma)]^2 + B \ \text{Tr}[(m^t \Sigma)^2] + C \ \text{Tr}(m^t \Sigma)\text{Tr}(m^* \Sigma^\dagger) + \text{H.c.}, \qquad (4)$$

where $\Sigma \in \text{U}(3)$ denotes the chiral fields for the 9 mesons. Note that the Nambu-Goldstone boson associated with the baryon number symmetry remains exactly massless, and thus does not appear in the mass term.

The coefficients A, B and C can be determined either by matching the vacuum energy of (4) with that computed in the microscopic theory [8] or by integrating out quark fields in the effective Lagrangian of quarks and the mesons [7]. Here we take the latter approach. Using the global color-chiral-axial-baryon number symmetry of dense QCD and the pattern of the diquark condensates (2) one can write an effective chiral Lagrangian for the quarks and the Nambu-Goldstone bosons as

$$\mathcal{L} = i\bar{\chi}_i^a \bar{\sigma}^\nu \partial_\nu \chi_i^a + \mu \bar{\chi}_i^a \bar{\sigma}^0 \chi_i^a + i\bar{\varphi}_i^a \bar{\sigma}^\nu \partial_\nu \varphi_i^a - \mu \bar{\varphi}_i^a \bar{\sigma}^0 \varphi_i^a - [m_{ij}\chi_i^a \varphi_j^a + \text{H.c.}]$$
$$+ \left[\chi_i^a (\Delta_\chi^\dagger)_{ij}^{ab} \chi_j^b + \varphi_i^a (\Delta_\varphi)_{ij}^{ab} \varphi_j^b + \text{H.c.} \right] + \mathcal{L}_{\text{NG}}(U, V), \qquad (5)$$

where

$$(\Delta_\chi^\dagger)_{ij}^{ab} = k_1 U_i^{*a} U_j^{*b} + k_2 U_j^{*a} U_i^{*b},$$
$$(\Delta_\varphi)_{ij}^{ab} = -[k_1 V_i^a V_j^b + k_2 V_j^a V_i^b]. \qquad (6)$$

The unitary matrices U and V denote the nonlinearly realized Nambu-Goldstone bosons arising from the symmetry breaking $SU_c(3) \times SU_L(3) \times U_{A+B}(1) \rightarrow SU_{c+L}(3)$ through the $\chi\chi$ condensation and $SU_c(3) \times SU_R(3) \times U_{B-A}(1) \rightarrow SU_{c+R}(3)$ through the $\bar{\varphi}\bar{\varphi}$ condensation, respectively. The $\mathcal{L}_{\text{NG}}(U, V)$ is the usual chiral Lagrangian for the Nambu-Goldstone bosons alone [12,13]. We also note that with (6) it was assumed, for simplicity, the gaps for antiquarks are identical to those for quarks [7]. It can be easily seen that the most important interactions for our calculation, those between the quarks and the Nambu-Goldstone bosons, satisfy the Goldberger-Treiman relation. This Lagrangian can be regarded as an effective

Lagrangian before color is gauged for the quarks and the 18 Nambu-Goldstone bosons U, V. Upon gauging color it can be seen that 8 out of the 18 Nambu-Goldstone bosons are eaten by the gluons via Higgs mechanism and there remain 10 color-singlet mesons as low energy excitations.

Integrating out the quark fields, which corresponds to the evaluation of quark one-loop diagrams in the background of constant U, V fields with two Dirac mass insertions (see Fig. 1), we obtain the mass term (4) in which Σ is given by

$$\Sigma = UV^{\dagger} \tag{7}$$

and

$$
\begin{aligned}
A = i \int \frac{d^4p}{(2\pi)^4} & \left\{ (k_1^2 + k_2^2) I_{8-}(p) I_{8+}(p) + k_1(k_1 + k_2/3)[I_{8-}(p) I_{8+}(p) \right. \\
& - I_{1+}(p) I_{8-}(p)] + k_1(k_1 + k_2/3)[I_{8-}(p) I_{8+}(p) - I_{1-}(p) I_{8+}(p)] \\
& + (k_1 + k_2/3)^2 [I_{8-}(p) I_{8+}(p) - I_{1-}(p) I_{8+}(p) - I_{1+}(p) I_{8-}(p) \\
& \left. + I_{1-}(p) I_{1+}(p)] \right\}, \\
B = i \int \frac{d^4p}{(2\pi)^4} & \left\{ 2k_1 k_2 I_{8-}(p) I_{8+}(p) + k_2(k_1 + k_2/3)[I_{8-}(p) I_{8+}(p) \right. \\
& \left. - I_{1+}(p) I_{8-}(p)] + k_2(k_1 + k_2/3)[I_{8-}(p) I_{8+}(p) - I_{1-}(p) I_{8+}(p)] \right\}, \\
C = -i\frac{1}{9} \int \frac{d^4p}{(2\pi)^4} & [-(p_0 - \mu)^2 + |\vec{p}|^2] \left\{ I_{8-}(p) I_{8+}(p) - I_{1-}(p) I_{8+}(p) \right. \\
& \left. - I_{1+}(p) I_{8-}(p) + I_{1-}(p) I_{1+}(p) \right\}, \tag{8}
\end{aligned}
$$

where

$$I_{1\mp}(p) = 1/[-p_0^2 + (|\vec{p}| \mp \mu)^2 + m_1^2], \qquad I_{8\mp}(p) = 1/[-p_0^2 + (|\vec{p}| \mp \mu)^2 + m_8^2]. \tag{9}$$

Here m_1 and m_8 are the Majorana masses for the singlet and octet quarks, respectively, under the unbroken $\mathrm{SU}_{L+R}(3)$ and are given as

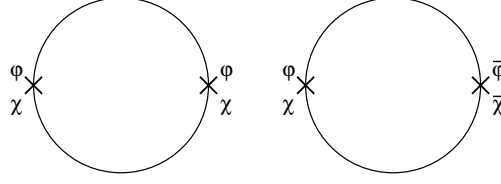$$m_1^2 = (3k_1 + k_2)^2, \quad m_8^2 = k_2^2. \tag{10}$$

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FIG. 1. Diagrams to be evaluated in the constant background of the chiral fields U and V.

Generally the diquark condensates depend on energy, but here we shall ignore this fact and treat k_i as constants. Then the integration over the loop momentum can be easily performed by doing contour integration over p_0 first and then replacing $d^3p \to 4\pi\mu^2 d|\vec{p}|$ for the integration over the spatial components. It can be easily seen that A and B arise from the first diagram in Fig.1 with $A, B \propto \Delta^2 \ln(\mu^2/\Delta^2)$, while C, coming from the second diagram, is given by $C \propto \Delta^4/\mu^2 \ln(\mu^2/\Delta^2)$, where $\Delta \sim k_i$. Note that C is suppressed by a factor Δ^2/μ^2 compared to A, B.

To determine the true chiral vacuum at a given chemical potential we have to minimize the potential $V(\Sigma) = -\mathcal{L}_m(\Sigma)$, with $\mathcal{L}_m(\Sigma)$ given by (4) and (8). To demonstrate a non-trivial vacuum alignment, we shall now take a simplified form for the quark mass matrix, in which the up and down quark masses are identical, as

$$m = m_d \operatorname{Diag}(1, 1, \delta), \;\; \delta \equiv m_s/m_d. \tag{11}$$

With this quark mass matrix it is easy to see that the Σ that minimizes the potential must be of the form

$$\Sigma = \operatorname{Diag}(\alpha, \alpha, \beta) \;\text{with}\; |\alpha|^2 = |\beta|^2 = 1, \tag{12}$$

where α, β are complex variables.

Substituting (11) and (12) into (4) we obtain

$$V = -2m_d^2 \left\{ \operatorname{Re}[A(2\alpha + \beta\delta)^2 + B(2\alpha^2 + \beta^2\delta^2)] + C\,|2\alpha + \beta\delta|^2 \right\}, \tag{13}$$

which can also be written as

$$V = -8m_d^2 \left\{ (2A + B)\cos^2\theta + (A + B)\delta^2/2\cos^2\phi + A\delta\cos(\theta + \phi) + C\delta\cos(\theta - \phi) \right\} \quad (14)$$

by putting $\alpha = \exp(i\theta)$ and $\beta = \exp(i\phi)$.

We first notice that the potential is symmetric under the transformation $(\alpha, \beta) \rightarrow (-\alpha, -\beta)$, so the minima of the potential must occur in pairs. Secondly we observe that for arbitrary coefficients A, B, and C the potential is stationary when θ, ϕ satisfy

$$\sin(\theta \pm \phi) = 0, \ \ \sin(2\theta) = 0, \ \ \sin(2\phi) = 0, \quad (15)$$

which have 8 common solutions corresponding to the following (α, β) pairs

$$(-i, -i), (-1, 1), (1, 1), (i, -i) \quad (16)$$

and their partners of opposite sign. Of course none of these pairs needs necessarily minimize the potential, but it can be shown numerically that the minima occur always on one of these solutions. This shows that the true chiral vacuum can align only to one of these eight directions.

In ideal situation we may know the dependence of the gap parameters k_i on μ, could choose the true chiral vacuum at a given chemical potential from the above discrete vacua, and investigate vacuum transitions as the chemical potential evolves. However, presently there is no reliable calculation to determine the gaps as functions of the chemical potential except when the chemical potential is extremely large [14], in which case Shwinger-Dyson equation can be used as an approximate gap equation and solved [15–17]. Even in this case the absolute magnitudes of the gaps are yet to be determined, but, it may not be so unreasonable to assume that k_i/μ are in the order of 0.1 at large chemical potential [5,16,6].

Taking into account this uncertainty we shall here treat k_i as free parameters and study the vacuum transitions as k_i vary. For definiteness, we shall put $\delta = 15$ and scan numerically the minima of the potential in the k_i parameter space defined by $0 \leq k_1/\mu \leq 0.5$ and $-0.5 \leq k_2/\mu \leq 0.5$. Note that k_1 can always be assumed positive, since its phase can be rotated away by the baryon number symmetry of the Lagrangian (5).

The result of the numerical scanning is shown in Fig. 2. As we see, the parameter space is divided into four domains according to their vacuum directions. In this figure the corresponding vacua of opposite sign to those in the group (16) are not included. The reason for this is that the numerical scanning shows there is always a potential barrier between a vacuum in the group (16) and the partners of those in the group (16), so in infinite volume limit the transitions between the vacua (16) and their partners are negligibly small and can be ignored. Therefore, only the transitions within the group of vacua in (16) need be considered.
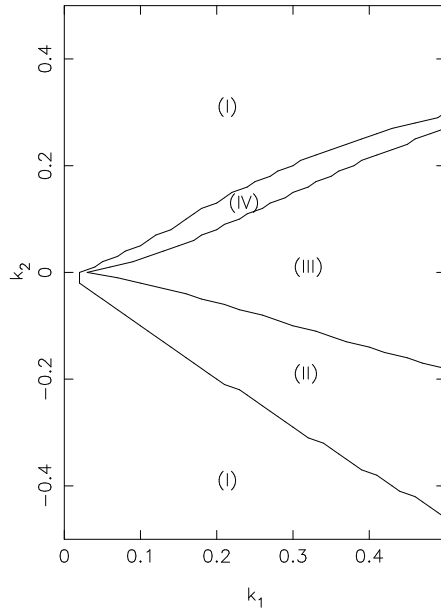

FIG. 2. Gap parameter space is divided into domains according to their vacua. Each domain is associated with a unique vacuum. k_i are in the unit of $\mu = 1$.

Because the vacuum can align only to a discrete number of directions the transitions between vacua will be of first order phase transition. To understand what happens at the transition, we look more carefully at the potential on the boundaries between the domains. For convenience, we shall designate the domains associated with the vacua defined in (16) by (I),(II),(III), and (IV), respectively.

First, consider the transition between the domain (I) and (II). As one approaches the boundary from either side of (I) and (II) it can be shown numerically that a potential valley opens up in (θ, ϕ) space along the line $\phi = -\theta - \pi$, with the bottom of the valley connecting

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the two points $(-\pi/2, -\pi/2)$ and $(-\pi, 0)$. When exactly on the boundary, we have from $V(\Sigma_\mathrm{I}) = V(\Sigma_\mathrm{II})$, with $\Sigma_\mathrm{I,II}$ defined by (12) and (16),

$$(2A + B) + (A + B)\delta^2/2 - 2C\delta = 0. \tag{17}$$

Then it is easy to see that on the bottom of the valley (i.e. along the direction $\phi = -\theta - \pi$) the potential is constant with $V = 8m_d^2(A - C)\delta$. Thus, in this case there will be no latent heat released at the phase transition.

Similarly, for the transition between (II) and (III) it can be shown that a valley opens up along the line $\phi = 0$ as the boundary is approached, and that the relation

$$A + C = 0, \tag{18}$$

holds on the boundary. On the bottom of the valley the potential is given by

$$V = -8m_d^2\left\{(2A + B)\cos^2\theta + (A + B)\delta^2/2\right\}, \tag{19}$$

which shows a barrier between the two vacua. This then indicates that there will be latent heat released at the phase transition. This may have some important phenomenological consequence in dense systems.

In a similar fashion we can easily show that for the transition across the domain (III) and (IV) a potential valley opens up along the direction $\phi = -\theta$, and on the bottom of the valley the potential is constant with $V = -8m_d^2(A - C)\delta$. In this case there will be no latent heat released as in the transition between (I) and (II). Also for the transition between (I) and (IV) a valley opens up along $\phi = -\pi/2$ direction and on the bottom of the valley the potential is given by

$$V = -4m_d^2(A + B)\delta^2\cos^2\theta, \tag{20}$$

which shows a barrier between the two vacua, and consequently, nonzero latent heat at the transition.

Could the phase transition of this kind occur when the chemical potential increases from zero to an asymptotic value? Although it is difficult to answer this question conclusively until

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we have the quark-mass induced meson potential at an arbitrary value of μ, which at smaller μ would probably contain the instanton-induced potential that gives mass to the η' meson and the old $\text{Tr}(m\Sigma)$ as well as the quark-mass quadratic $\mathcal{L}_m(\Sigma)$ in (4), there is an interesting observation concerning this question. It is well known that at large chemical potential the sextet components of the condensates (2) is suppressed [18,19], thus $k_1 \approx -k_2$, which then suggests the system must be in domain (I) at high density. The vacuum associated with domain (I) is $\Sigma_0 = \text{Diag}(-i, -i, -i)$ that has an overall factor $-i$ compared to the vacuum $\Sigma_0 = I$ at zero density. Although this does not imply a first order phase transition of the kind hitherto discussed, at least it does suggest that the vacuum must shift from the unit matrix at zero density to something else as the chemical potential increases.

This also leads to an interesting consequence of the vacuum alignment, namely that parity must be spontaneously broken at high density. The possibility of parity violation in dense QCD was pointed out by Pisarski and Rischke [4], and noted also in [20,21,2], who observed that the axial U(1) is almost a good symmetry at large μ due to the suppression of instanton effects, so a vacuum with no parity symmetry can be as easily formed as the parity even vacuum. Since the axial U(1) is never exactly restored this possibility, however, cannot be realized with massless quarks. The lowest energy state is always the parity even vacuum. However, when quarks are massive, the explicit breaking of the axial U(1) by quark masses can be more important than the small anomalous breaking, so the true vacuum could break parity.

Indeed, we see that this possibility is realized through the chiral vacuum alignment. Since the diquark condensates satisfy (3) in parity even vacuum it can be seen using (7) that only the vacuum $\Sigma_0 = I$ associated with the domain (III) is parity even. Therefore, when the system is in domain (I), (II), and (IV) parity is spontaneously broken. Moreover, since the system should be in domain (I) as $\mu \to \infty$, parity must be broken at high density.

To conclude, we have studied quark-mass induced chiral vacuum alignment in cold, dense QCD. When quarks are massless any point in the continuum of chiral vacua can be chosen as the vacuum, but when nonzero quark masses are introduced, the true vacuum must be

found via Dashen's procedure. We have shown that in color-flavor locking phase at large chemical potential the vacuum can align only to a discrete number of directions, and the nature of the vacuum transitions is of first order with vanishing or nonvanishing latent heat. It was also shown that at high density parity is spontaneously broken through the vacuum alignment. The consequence of the first order phase transitions and the parity violation may become important in dense systems like quark stars and heavy ion collisions.

Finally, we remark on the calculation of the meson masses. Usually in meson mass calculation it was assumed that the vacuum is at $\Sigma_0 = I$, and the meson potential was expanded around this vacuum to pick up the meson spectrum. However, as we have seen, the vacuum is not necessarily at the unit matrix and so this is not always correct. For correct meson masses one should first find the true vacuum and then expand the potential about it.

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